Framework for High School Mathematics

High school mathematics standards fall into six conceptual categories which are covered to varying degrees throughout all of the mathematics courses a student takes during his or her high school career. Each conceptual category can be further broken down into domains and then into subcategories. Every course addresses each of the six conceptual categories but will not necessarily include every domain or subcategory.

The six conceptual categories and their abbreviations are:

1. Number and Quantity (N)
2. Algebra (A)
3. Functions (F)
4. Modeling (M)
5. Geometry (G)
6. Statistics and Probability (S)

In addition to the standards needed by all students, additional mathematical standards may be needed by students planning to take advanced courses in calculus, statistics, or discrete mathematics. These additional standards are noted in the curriculum with a plus symbol (+).

Some standards or headings for groups of standards may include an asterisk (*). If the asterisk is in a group heading it applies to all standards under that heading. The asterisk indicates that the standard requires modeling. Modeling links the classroom standards to real life including everyday skills, work, and decision making.

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Conceptual Category: Number and Quantity

Numbers

As students progress through the eighth grade their knowledge of numbers grows from merely using counting numbers (1, 2, 3, ...), to including zero, to working with fractions. In middle school they begin working with negative numbers and irrational numbers. In high school, the concept of imaginary numbers will be added to the real numbers to form complex numbers. As the number system grows, the meanings of addition, subtraction, multiplication, and division are extended and they remain true under the commutative, associative, and distributive properties.

Calculators, spreadsheets, and computer algebra systems allow students to explore the growing number systems and their notation. They can be used to generate data for numerical experiments; to help understand the workings of matrix, vector, and complex number algebra, and to experiment with non-integer exponents.

Quantities

Real world measurement through eighth grade typically involves length, area, and volume. High school adds a variety of units through modeling including acceleration, currency conversions, and derived quantities like person-hours, heading degree days, per-capita income, and rates in everyday life. Quantification, which will become important in science, is also introduced as students are required to analyze real-life situations and then find solutions for the problems encountered. An example of quantification would be proposing measures to collect data regarding annual highway fatalities. This type of reasoning is important in both government and private agencies which must conceptualize needs and then create solutions for them.

The Conceptual Category Numbers and Quantity (N) includes the following four Domains and their related subcategories.

The Real Number System (N-RN)

- Extend the properties of exponents to rational exponents
- Use the properties of rational and irrational numbers

Quantities (N-Q)

- Reason quantitatively and use units to solve problems

The Complex Number System (N-CN)

- Perform arithmetic operations with complex numbers
- Represent complex numbers and their operations on the complex plane
- Use complex numbers in polynomial identities and equations

Vector and Matrix Quantities

- Represent and model with vector quantities
- Perform operations on vectors
- Perform operations on matrices and use matrices in applications

Expressions
An expression is a record of computation. Recognized conventions regarding the use of grouping symbols and application of the order of operation ensures that expressions are explicit and understood.

Equations and Inequalities
Equations are statements of equality between two expressions. The solution for an equation is the value(s) of the variables which make the equation true. The solutions of an equation in one variable form a set of numbers. The solutions of an equation in two variables form a set of ordered pairs which can be plotted on the coordinate plane. Two or more equations and/or inequalities for a system. Strategic competence in solving equations and inequalities requires students to be able to look ahead for productive manipulations and the ability to anticipate the type and number of solutions.

Connections to Functions and Modeling
Converting a verbal description to an equation, inequality, or system is essential to modeling real-world problems.

The Conceptual Category Algebra (A) includes the following four domains and their related subcategories:

Seeing Structure in Expressions (A-SSE)
- Interpret the structure of expressions
- Write expressions in equivalent forms to solve problems

Arithmetic With Polynomials and Rational Expressions (A-APR)
- Perform arithmetic operations on polynomials
- Understand the relationship between zeros and factors of polynomials
- Use polynomial identities to solve problems
- Rewrite rational expressions

Creating Equations (A-CED)
- Create equations that describe numbers or relationships

Reasoning With Equations and Inequalities (A-REI)
- Understand solving equations as a process of reasoning and explain the reasoning
- Solve equations and inequalities in one variable
- Represent and solve equations and inequalities graphically

Conceptual Category: Functions

Functions are essential in the construction of mathematical models. They describe situations where one quantity determines another. Two important types of functions are linear and exponential functions. The relationships described by functions make them valuable in modeling everyday events.

The Conceptual Category Functions (F) includes the following four domains and their related subcategories:

Interpreting Functions (F-IF)

- Understand the concept of a function and use function notation
- Interpret functions that arise in applications in terms of the context
- Analyze functions using different representations

Building Functions (F-BF)

- Build a function that models a relationship between two quantities
- Build new functions from existing functions

Linear, Quadratic, and Exponential Models (F-LE)

- Construct and compare linear, quadratic, and exponential models and solve problems
- Interpret expressions for functions in terms of the situation they model

Trigonometric Functions (F-TF)

- Extend the domain of trigonometric functions using the unit circle
- Model periodic phenomena with trigonometric functions
- Prove and apply trigonometric identities

Conceptual Category: Modeling

Modeling links mathematics from the classroom with everyday life, work, and decision making. It is the process of choosing the appropriate mathematics and statistics to analyze and understand situations and then make informed decisions. Examples of situations which solutions require modeling include:

- Estimating how much water and food is needed for emergency relief in a devastated city of 3 million people and how it might be distributed
- Planning a table tennis tournament for 7 players using 4 tables, where each player plays against every other player
- Designing the layout of the stalls in a school fair so as to raise as much money as possible
- Analyzing stopping distance for a car
- Modeling savings account balance, bacterial colony growth, or investment growth
- Engaging in critical path analysis such as the turnaround space required for an aircraft at an airport
- Analyzing risk in situations such as extreme sports, pandemics, and terrorism
- Relating population statistics to individual problems

The basic modeling cycle is as follows

1. Identify the variables and select those that represent essential features
2. Formulate a model by creating and selecting geometric, graphical, tabular, or statistical representations to describe relationships between the variables
3. Analyze and perform operations, draw conclusions
4. Interpret results in terms of the original situation
5. Validate conclusions and either improve the model (return to step 2) or move forward
6. Report on conclusions and the reasoning behind them

Descriptive modeling describes and summarizes phenomena. Graphs can be helpful.

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Conceptual Category: Geometry

School mathematics focuses primarily on plane Euclidean geometry. High school students solidify their understanding of geometry through precise definitions and by developing proofs. Analytic geometry connects algebra and geometry to create powerful problem solving tools.

The Conceptual Category Geometry (G) includes the following six domains and their related subcategories:

Congruence (C-CO)

- Experiment with transformations in the plane
- Understand congruence in terms of rigid motions
- Prove geometric theorems
- Make geometric constructions

Similarity, Right Triangles, and Trigonometry (G-SRT)

- Understand similarity in terms of similarity transformations
- Prove theorems involving similarity
- Define trigonometric ratios and solve problems involving right triangles
- Apply trigonometry to general triangles

Circles (G-C)

- Understand and apply theorems about circles
- Find arc lengths and areas of sectors of circles

Expressing Geometric Properties with Equations (G-GPE)

- Translate between the geometric description and the equation for a conic section
- Use coordinates to prove simple geometric theorems algebraically

Geometric Measurement and Dimension (G-GMD)

- Explain volume formulas and use them to solve problems
- Visualize relationships between two-dimensional and three-dimensional objects

Modeling with Geometry (G-MG)

- Apply geometric concepts in modeling situations

Conceptual Category: Statistics and Probability*

Statistics provides tools to describe the variability in data so it can be taken into account when making decisions or predictions. This involves finding patterns and deviations from patterns.

The Conceptual Category Statistics and Probability (S) includes the following four domains and their related subcategories:

Interpreting Categorical and Quantitative Data (S-ID)

- Summarize, represent, and interpret data on a single count or measurement variable
- Summarize, represent and interpret data on two categorical and quantitative variables
- Interpret linear models

Making Inferences and Justifying Conclusions (S-IC)

- Understand and evaluate random processes underlying statistical experiments
- Make inferences and justify conclusions from sample surveys, experiments, and observational studies

Conditional Probability and the Rules of Probability (S-CP)

- Understand independence and conditional probability and use them to interpret data
- Use the rules of probability to compute probabilities of compound events in a uniform probability model

Using Probability to Make Decisions (S-MD)

- Calculate expected values and use them to solve problems
- Use probability to evaluate outcomes of decisions